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STABILITY OF A PLASMA ROTATING IN CROSSED ELECTRICAL AND MAGNETIC FIELDS

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16. Abstract

It is shown that, in addition to high-frequency longitudinal oscillations, oscillations at a frequency of the order of the ion cyclotron frequency can arise (in the presence of crossed fields) in a rotating cylindrical plasma with different angular velocities of the electrons and ions. The stability analysis is performed in hydrodynamic approximation.

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STABILITY OF A PLASMA ROTATING IN CROSSED ELECTRICAL AND MAGNETIC FIELDS

V.V. Dolgopolov, V.L. Sizonenko, and K.N. Stepanov

Introduction /18*

Electrons and ions rotate with different angular velocities in a plasma cylinder in longitudinal magnetic and radial electric fields, owing to the presence of centrifugal forces. The relative movement of electrons and ions may be the cause of formation of various instabilities in such a plasma. In this case, both small-scale longitudinal oscillations of the plasma, for which plasma inhomogeneities are insignificant, and oscillations of the drift wave type and channeled disturbances can be excited [1].

These instabilities lead to plasma turbulence and its heating. High-frequency turbulent noise in the plasma, the frequencies of which considerably exceed the ion cyclotron frequency, were discovered experimentally [2-5]. This instability also was studied experimentally in works [6-8], in which excitation of powerful ion cyclotron oscillations, accompanied by strong heating of the ion component, were discovered and investigated, in addition. Turbulent heating of the ion component and turbulent, high-frequency pulses in the rotating plasma also were studied experimentally in works [9-10]. Evidently, observed heating of ions in an ion magnetron can be explained by the presence of a similar type of instability [11].

The stability of a rotating plasma cylinder is investigated in this work, and it is shown that, along with high-frequency longitudinal oscillations in such a plasma, excitation of oscillations, with frequencies and increments on the order of the ion cyclotron frequency is possible.

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*Numbers in the margin indicate pagination in the foreign text.

1. Short Wave Oscillations

In the equilibrium state, the distribution function of α-type particles is determined from the equation

$$v_r \frac{\partial f_{0\alpha}}{\partial r} + \frac{v_\varphi}{r} \frac{\partial f_{0\alpha}}{\partial \varphi} + \frac{e_\alpha}{m_\alpha} E_0 \frac{\partial f_{0\alpha}}{\partial v_r} - \omega_{H\alpha} \frac{\partial f_{0\alpha}}{\partial \Phi} = 0, \tag{1.1}$$

where $E_0(r)$ is the voltage of the radial electric field, $\omega_{H\alpha} = e_\alpha H_0/m_\alpha c$ is the gyrofrequency of α-type particles, ϕ and Φ are the azimuthal angles in normal space and in velocity space, respectively, and $v_r = v_\perp \times \cos(\Phi - \phi)$, $v_\phi = v_\perp \sin(\Phi - \phi)$.

If the Larmor radius of the particles is considerably less than the radius of the plasma, the first term in Eq. (1.1) can be disregarded and v_ϕ/r can be replaced by the angular velocity of rotation ω_α. Then, with ω_α independent of r, we obtain the following expression from (1.1): /19

$$f_{0\alpha} = n_0 \left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{3/2} \exp\left\{-\frac{m_\alpha}{2T_\alpha} [v_r^2 + (v_\varphi - r\omega_\alpha)^2 + v_z^2]\right\}. \tag{1.2}$$

In this case, the value of the rotational velocity $u_\alpha = r\omega_\alpha$ is determined from the equilibrium conditions

$$\frac{u_\alpha^2}{r} = \frac{e_\alpha}{m_\alpha} E_0 + u_\alpha \omega_{H\alpha} - \frac{1}{m_\alpha n_0} \frac{\partial p_\alpha}{\partial r}, \tag{1.3}$$

where $p_\alpha = n_0 T_\alpha$.

Let us examine the case of short wave oscillations, when plasma inhomogeneities can be disregarded. (For this, the longitudinal wave number $k_{||}$ must not be very small.) We shall look for a solution for a field disturbance $E = -\nabla\psi$ and the distribution function $\tilde{f_\alpha} = f_{0\alpha} g_\alpha$, in the form

$$\psi, g_\alpha \sim \exp\left\{i\left[k_\parallel z + m\varphi + \int k_r dr - \omega t\right]\right\}, \tag{1.4}$$

where m is a whole number. Then, disregarding values connected with the density gradient, we obtain the following equation for g_α:

$$i(k_\parallel v_z - \omega + m\omega_\alpha + k_\perp v'_\perp \cos\vartheta) g_\alpha - (\omega_{H\alpha} + 2\omega_\alpha)\frac{\partial g_\alpha}{\partial \vartheta} = R_\alpha(\vartheta), \tag{1.5}$$

where

$$v'_\varphi = v_\varphi - r\omega_\alpha = v'_\perp \sin\vartheta',\ v_r = v'_\perp \cos\vartheta',\ \vartheta = \vartheta' - \beta,\ k_r = k_\perp \cos\beta,$$

$$k_\varphi = \frac{m}{r} = k_\perp \sin\beta,\ R_\alpha(\vartheta) = -i\frac{e_\alpha}{T_\alpha}\psi(k_\perp v'_\perp \cos\vartheta + k_\parallel v_z).$$

If the replacement of variables $v_\perp' \to v_\perp$ is made in Eq. (1.5),

$$\vartheta' \to \Phi - \varphi,\quad \omega - m\omega_\alpha \to \omega,\quad \omega_{H\alpha} + 2\omega_\alpha \to \omega_{H\alpha}, \tag{1.6}$$

it coincides with the equation for the oscillator portion of the distribution function in a quiescent plasma.

Integrating Eq. (1.5) and substituting the expression obtained for $\tilde{f_\alpha} = f_{0\alpha} g_\alpha$ in the Poisson equation, we obtain the following dispersion equation:

$$1 + \sum_\alpha \delta\varepsilon_\alpha(\omega) = 0, \tag{1.7}$$

where

$$\delta\varepsilon_\alpha = \frac{\omega_{p\alpha}^2}{k^2 v_{T\alpha}^2}\left[1 + i\sqrt{\pi} z_{0\alpha} \sum_{n=-\infty}^{\infty} A_n(\mu_\alpha) w(z_{n\alpha})\right],\quad \mu_\alpha = \frac{k_\perp^2 v_{T\alpha}^2}{(\omega_{H\alpha} + 2\omega_\alpha)^2},$$

$$k_\perp^2 = k_r^2 + k_\varphi^2, \tag{1.8}$$

$$z_{n\alpha} = \frac{\omega - m\omega_\alpha - n(\omega_{H\alpha} + 2\omega_\alpha)}{\sqrt{2} k_\parallel v_{T\alpha}},\ A_n(x) = e^{-x} I_n(x),$$

$$w(z) = e^{-z^2}\left(\frac{k_\parallel}{|k_\parallel|} + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right).$$ /20

Dispersion equation (1.7), of course, has the same form as for a uniform plasma with a transverse current [12, 13], and it is only necessary to carry out replacements (1.6). The value $u_\phi = r\omega_\alpha$ plays the part of transverse velocity, and the value $k_\phi = m/t$, the part of azimuthal component of the wave vector. Therefore, in those cases when effects connected with the extremity of the Larmor radius do not play a part, instabilities in the rotating plasma are similar to instabilities in a uniform plasma with a transverse current, and the expressions presented in works [12-15] can be used for the frequencies and growth increments of the oscillations. However, the frequencies and growth increments of oscillations and instability threshold in a rotating plasma can differ significantly from those in a uniform plasma, for oscillations for which the extremities of the Larmor radius are significant; this refers especially to the case of resonance $\omega_{H\alpha} \simeq -2\omega_\alpha$.

Let us investigate the dispersion equation in a series of limiting cases. Here, we will assume everywhere that the oscillation frequencies are considerably less than the electron gyrofrequencies and, moreover, that $|\omega_{He}| \gg k_\parallel v_{Te}$. In this case, it can be assumed that

$$\delta\varepsilon_e = \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\,[1 + i\sqrt{\pi}z_{0e}w(z_{0e})\,A_0(\mu_e)]. \tag{1.9}$$

a) Let us examine hydrodynamic instabilities, for which $\mu_\alpha \ll 1$ and $|z_{n\alpha}| \gg 1$. In this case, Eq. (1.7) takes the form

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2\cos^2\Theta}{(\omega - m\omega_e)^2} - \sum_{\alpha=i}\frac{\omega_{p\alpha}^2}{(\omega - m\omega_\alpha)^2 - (\omega_{H\alpha} + 2\omega_\alpha)^2} = 0, \tag{1.10}$$

where $\cos^2\Theta = k_{\|}^2/(k_{\perp}^2 + k_{\|}^2) \ll 1$.

If $\omega_{pi} \gtrsim \omega_{Hi}$, Eq. (1.10), with $\cos^2\Theta \sim m_e/m_i$, has the solution

$$\begin{gathered}\mathrm{Re}\,\omega - m\omega_i \sim \mathrm{Im}\,\omega \sim m\,|\omega_e - \omega_i| \sim \omega_h, \\ \omega_h = \frac{\omega_{pi}}{\sqrt{1 + \frac{\omega_{pe}^2}{\omega_{He}^2}}} \sim \min\left(\omega_{pi}, \sqrt{\omega_{Hi}\,|\omega_{He}|}\right).\end{gathered} \tag{1.11}$$

where

The frequency and growth increment of these oscillations is considerably larger than the ion cyclotron frequency, so that the ions can be considered unmagnetized. In this case, dispersion equation (1.10) can be used and, with $\mu_i \gtrsim 1$, it is only necessary to disregard $(\omega_{Hi} + 2\omega_i)^2$ in comparison with $(\omega - m\omega_i)^2$.

We note that, in the presence of ions of two or more kinds, moving relative to one another with velocity $u = r|\omega_{i1} - \omega_{i2}|$, excitation of ion-ion cluster instabilities is possible, with frequencies on the order of

$$\mathrm{Re}\,\omega \sim \mathrm{Im}\,\omega \sim |m(\omega_{i1} - \omega_{i2})| \sim \omega_{pi}\left(\cos^2\Theta < \frac{m_e}{m_i}\right),$$

in which electrons do not take part in these oscillations.

If $\cos^2\Theta \gg m_e/m_i$ the term dependent on ions in (1.10) is /21 small in comparison with terms dependent on electrons, only if ω is not close to $m\omega_i$. Assuming $\omega = m\omega_i + \Delta\omega$, where $|m(\omega_e - \omega_i)| \gg |\Delta\omega| \gg \omega_{Hi}$, we obtain

$$\Delta\omega = \pm i \frac{\omega_{pi}}{\sqrt{\frac{\omega_{pe}^2 \cos^2\Theta}{m^2(\omega_e-\omega_i)^2} - 1 - \frac{\omega_{pe}^2}{\omega_{He}^2}}}.$$

Under resonant conditions,

$$|m(\omega_e-\omega_i)| = \frac{\omega_{pe}\cos\Theta}{\sqrt{1+\frac{\omega_{pe}^2}{\omega_{He}^2}}},$$

the growth increment reaches a maximum value:

$$\Delta\omega = \frac{\pm 1 + i\sqrt{3}}{2^{4/3}} \frac{(\omega_{pi}^2\omega_{pe}\cos\Theta)^{1/3}}{\left(1+\frac{\omega_{pe}^2}{\omega_{He}^2}\right)^{1/2}}.$$

In the case of very low density ($\omega_{pi} \lesssim \omega_{Hi}$), oscillations propagating at an angle Θ to the magnetic field, which is not close to $\pi/2$, also can be unstable. In this case, dispersion equation (1.10) should be replaced by the equation

$$1 - \frac{\omega_{pe}^2\cos^2\Theta}{(\omega - m\omega_e)^2} - \sum_{\alpha=i}\left[\frac{\omega_{p\alpha}^2\cos^2\Theta}{(\omega-m\omega_i)^2} + \frac{\omega_{p\alpha}^2\sin^2\Theta}{(\omega-m\omega_i)^2-(\omega_{H\alpha}+2\omega_\alpha)^2}\right] = 0. \quad (1.12)$$

Its solution is close to $m\omega_i$:

$$\omega = m\omega_i + \Delta\omega \quad (|\Delta\omega| \ll |m(\omega_e-\omega_i)|), \quad (1.13)$$

where

$$\Delta\omega = \pm i \frac{\omega_{pi}\cos\Theta}{\sqrt{\left[\frac{\omega_{pe}\cos\Theta}{m(\omega_e-\omega_i)}\right]^2 - 1}}, \quad (1.14)$$

if $m(\omega_e - \omega_i)$ is not close to $\omega_{pe}\cos\Theta$, and

$$\Delta\omega = \frac{\pm 1 + i\sqrt{3}}{2^{4/3}}\,\omega_{pe}\cos\Theta\left(\frac{m_e}{m_i}\right)^{1/3}, \tag{1.15}$$

if $m(\omega_e - \omega_i) = \omega_{pe}\cos\Theta$. In obtaining these expressions, we assume that $|\Delta\omega| << |\omega_{Hi} + 2\omega_i|$.

In addition, this equation has still another solution, close to $\omega = m\omega_i \pm (\omega_{Hi} + 2\omega_i)$. One of them corresponds to unstable oscillations, if $m\omega_i \pm |\omega_{Hi} + 2\omega_i| \sim \omega_{pe}\cos\Theta + m\omega_e$. In this case, $\mathrm{Im}\,\omega = \gamma$, where

$$\gamma^2 = \mp \frac{\omega_{pi}^2 \sin^2\Theta}{4}\,\frac{\omega_{pe}\cos\Theta}{\omega_{Hi} + 2\omega_i}. \tag{1.16}$$

We note that conditions for applicability of the hydrodynamic approximation for unmagnetized oscillations with frequencies (1.11) /22

$$|z_{0e}| \sim \left|\frac{m(\omega_e - \omega_i)}{k_{\parallel} v_{Te}}\right| \gg 1,\quad |z_i| \equiv \left|\frac{\omega - m\omega_i}{\sqrt{2}kv_{Ti}}\right| \sim \frac{|m(\omega_e - \omega_i)|}{k_{\perp} v_{Ti}} \gg 1 \tag{1.17}$$

are satisfied, if the relative azimuthal velocity of the particles u is considerably greater than the thermal velocity of the ions and the velocity of sound.

b) We will consider the condition $u \gg v_{Ti}$ to be satisfied. Let us demonstrate that, in this case, there is a high-frequency kinetic instability, with frequency and growth increment (1.11), in a two-component plasma, with hot ions and cold electrons.

Since the ions are unmagnetized for high-frequency oscillations,

$$\delta\varepsilon_i = \frac{\omega_{pi}^2}{k^2 v_{Ti}^2}\left[1 + i\sqrt{\pi} z_i w(z_i)\right]. \tag{1.18}$$

Considering condition (1.17) to be satisfied for electrons, we can assume

$$\delta\varepsilon_e = \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 \cos^2\Theta}{(\omega - m\omega_e)^2}. \tag{1.19}$$

Substituting (1.18) and (1.19) in Eq. (1.7), and assuming $\omega = m\omega_i + i\gamma$, where $\gamma \ll |m(\omega_e - \omega_i)|$, we obtain

$$f(x) \equiv 1 - \sqrt{\pi} x e^{x^2} - 2e^{x^2}\int_0^x e^{-t^2} dt = \frac{k^2 v_{Ti}^2}{\omega_{pi}^2}\left[\frac{\omega_{pe}^2 \cos^2\Theta}{m^2(\omega_e - \omega_i)^2} - 1 - \frac{\omega_{pe}^2}{\omega_{He}^2}\right], \tag{1.20}$$

where $x = \frac{\gamma}{\sqrt{2} k v_{Ti}}$. The function $f(x)$ decreases monotonically from unity to zero, with increase in x from 0 to infinity. Therefore, dispersion equation (1.20) has a solution $x \sim 1$, if the right part of (1.20) is on the order of (but less than) unity. In this case, for $\gamma \sim k v_{Ti}$, we obtain an estimate of (1.11). From the condition that the right part of (1.20) is on the order of unity, we find that characteristic values of $\cos\Theta$ are equal, in order of magnitude, to

$$\cos\Theta \sim \frac{u}{v_{Ti}}\sqrt{\frac{m_e}{m_i}} \ll 1, \quad k \sim k_\varphi. \tag{1.21}$$

The conditions for applicability of the estimates obtained ($|z_{0e}| \gg 1$) are satisfied if $T_i \gg T_e$.

Dispersion equation (1.20) is not applicable under resonance conditions, when the right part of Eq. (1.20) returns to zero

$$|m(\omega_e-\omega_i)|=\frac{\omega_{pe}\cos\Theta}{\sqrt{1+\frac{\omega_{pe}^2}{\omega_{He}^2}}}. \tag{1.22}$$

In this case, using (1.18) and (1.19), we find that dispersion equation (1.7) takes on the form

$$z_i-z_0+\eta[1+i\sqrt{\pi}z_iw(z_i)]=0, \tag{1.23}$$

where /23

$$z_0=\frac{m^3(\omega_i-\omega_e)^3}{\sqrt{8}kv_{Ti}\omega_{pe}^2\cos^2\Theta}\left[\frac{\omega_{pe}^2\cos^2\Theta}{m^2(\omega_e-\omega_i)^2}-1-\frac{\omega_{pe}^2}{\omega_{He}^2}\right],\ \eta=\frac{\omega_{pi}^2m^3(\omega_i-\omega_e)^3}{\sqrt{8}k^3v_{Ti}^3\omega_{pe}^2\cos^2\Theta}.$$

From here we find that Re $z_i \sim$ Im $z_i \sim 1$, at $|z_0| \lesssim 1$ and $\eta \sim 1$, i.e.

$$\mathrm{Re}\,\omega-m\omega_i\sim\gamma\sim kv_{Ti}\sim\frac{(\omega_{pi}^2\omega_{pe}\cos\Theta)^{1/3}}{\left(1+\frac{\omega_{pe}^2}{\omega_{He}^2}\right)^{1/2}}. \tag{1.24}$$

Thus, under resonance conditions, the growth increment for the kinetic instability ($\gamma \sim kv_{Ti}$) is on the order of the maximum growth increment of hydrodynamic oscillations.

For applicability of expressions (1.23) and (1.24), it is necessary that the conditions $\mu_e = k^2\rho_e^2 \ll 1$ and $|z_{0e}| \gg 1$ be satisfied. These inequalities are satisfied if

$$\sqrt{\frac{T_i}{T_e}}\gg\left(\frac{u}{v_{Ti}}\right)^{1/3}.$$

c) In a strongly nonisothermal plasma, with hot electrons and cold ions ($T_e >> T_i$), with $u >> v_s = \sqrt{T_e/m_i}$, the development of high-frequency ($\gamma >> \omega_{Hi}$) oscillations of the ionic acoustic type is possible in a two-component plasma. In this case, $|z_i| >> 1$ and $\delta\varepsilon_i = -\frac{\omega_{pi}^2}{(\omega - m\omega_i)^2}$. Taking expression (1.9) into account for $\delta\varepsilon_e$, we obtain from (1.7)

$$\omega = m\omega_i \pm \frac{\omega_{pi}}{\sqrt{1 + \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left[1 + i\sqrt{\pi} z_{0e} w(z_{0e}) A_0(\mu_e)\right]}}, \tag{1.25}$$

where

$$z_{0e} = \frac{m(\omega_i - \omega_e)}{\sqrt{2} k_\parallel v_{Te}}.$$

With $|z_{0e}| \sim 1$, $k\rho_e \gtrsim 1$ and $\omega_{pe} \sim kv_{Te}$, in accordance with (1.25), in order of magnitude, we have

$$\gamma \sim \sqrt{\omega_{Hi}|\omega_{He}|}, \; \omega_{pe} \gtrsim |\omega_{He}|. \tag{1.26}$$

The conditions for applicability of expressions (1.25) and (1.26), $|z_i| >> 1$, are satisfied if $T_e >> T_i$.

d) In an isothermal plasma ($T_e \sim T_i$) of high density ($\omega_{pe} >> |\omega_{He}|$), kinetic instabilities, with growth increment $\gamma >> \omega_{Hi}$, also exist at $u >> v_{Ti}$. The ions can be considered unmagnetized for such oscillations, and expression (1.18) can be used for $\delta\varepsilon_i$, and expression (1.9) for $\delta\varepsilon_e$. In this case, $|z_{0e}| \sim |z_i| \sim 1$ for kinetic instabilities, so that electrons and ions with thermal velocities are resonant. The growth increments of these oscillations are determined in order of magnitude by relations (1.11).

Let us proceed now to study of oscillations, frequencies and growth increments, which are comparable to ion cyclotron frequencies.

e) Let us begin with examination of hydrodynamic instabilities in a low-density plasma. At $\omega_{pi} \lesssim \omega_{Ni}$, dispersion equation (1.10) has the solution /24

$$\mathrm{Re}\,\omega - m\omega_i \sim \gamma \sim m(\omega_i - \omega_e) \sim \omega_{Hi}. \tag{1.27}$$

f) Let us now examine oscillations with a finite ion Larmor radius ($\mu_i \gtrsim 1$). Assuming that $|z_{ni}| >> 1$, and using expression (1.9) for $\delta\varepsilon_e$, at $|z_{0e}| << 1$ and $\mu_e << 1$, we present dispersion equation (1.7) in the form

$$1 + k^2 r_{pi}^2 + \frac{T_i}{T_e} - \sum_{n=1}^{\infty} \frac{2(\omega - m\omega_i)^2 A_n(\mu_i)}{(\omega - m\omega_i)^2 - n^2(\omega_{Hi} + 2\omega_i)^2} - A_0(\mu_i) = -i\sqrt{\pi} z_{0e} \frac{T_i}{T_e} - i\sqrt{\pi} z_{0i} \sum_{n=-\infty}^{\infty} A_n(\mu_i) e^{-z_{ni}^2}. \tag{1.28}$$

Discarding small terms in the right part of this equation, we find the natural frequencies $\omega = \omega^{(j)}(\mu_i, m)$, corresponding to longitudinal ion cyclotron oscillations. Graphs of these frequencies vs. μ_i, at $\omega_i = 0$ have been presented in works [16, 17]. We can use the results of [16, 17] for determination of the frequencies $\omega^{(j)}(\mu_i, m)$, taking substitutions (1.6) into account.

Considering the smallness of the right side of (1.28), it is easy to find the growth increment of the cyclotron oscillations

$$\gamma = -\frac{\sqrt{\pi} z_{0e} \frac{T_i}{T_e} + \sqrt{\pi} z_{0i} \sum_{n=-\infty}^{\infty} A_n(\mu_i) e^{-z_{ni}^2}}{\sum_{n=-\infty}^{\infty} A_n(\mu_i) n (\omega_{Hi} + 2\omega_i)(\omega - m\omega_i - n\omega_{Hi} - 2n\omega_i)^{-2}}, \qquad (1.29)$$

where

$$z_{0e} = \frac{\omega^{(l)} - m\omega_e}{\sqrt{2} k_{\parallel} v_{Te}}, \quad z_{ni} = \frac{\omega^{(l)} - m\omega_i - n(\omega_{Hi} + 2\omega_i)}{\sqrt{2} k_{\parallel} v_{Ti}}.$$

The boundary of the instability region can be determined, assuming $\gamma = 0$. Using the results of [16], we find the critical value of the relative velocities of electrons and ions, above which excitation of cyclotron oscillations is possible, $u_{cr} \ll v_{Ti}$. However, it should be remembered that applicability of the expressions obtained above for the frequencies and growth increments of the high-frequency oscillations are limited by the conditions $\gamma \gg |\omega_{Hi} + 2\omega_i|$. At $T_i \gg T_e$, the maximum growth increment of the electron-acoustic oscillations

$$\gamma \approx 0{,}15 \sqrt{\omega_{Hi} |\omega_{He}|} \left(\frac{u}{v_{Ti}}\right)^2$$

becomes very small at $u \ll v_{Ti}$, and the inequality $\gamma \gg |\omega_{Hi} + 2\omega_i|$ can be destroyed. However, under resonance conditions $\omega_{Hi} \sim -2\omega_i$, this inequality will be satisfied, even at small u, so that the excitation threshold of electron-acoustical oscillations actually will be very low.

2. Stability of a Nonhomogeneous Rotating Plasma /25

Let us proceed now to investigations of disturbances, for which inhomogeneities of the plasma prove to be significant. In this case, we restrict ourselves to examination of oscillations, for which the presence of thermal movement of electrons is negligible, so that the hydrodynamic approximation can be used to describe them. (Instabilities of a rotating plasma, for description of which a kinetic analysis is necessary, were investigated at $k_{||} = 0$ in a number of works [18, 19 and others].

Linearizing the equations of motion and continuity equations for electrons and ions, we present the Poisson equation in the form [1]

$$\frac{1}{r}\frac{d}{dr}\left(r\varepsilon_1\frac{d\psi}{dr}\right)-\left(k_{\|}^2\varepsilon_3+k_{\varphi}^2\varepsilon_1+k_{\varphi}\frac{d\varepsilon_2}{dr}\right)\psi=0, \tag{2.1}$$

where

$$\varepsilon_1 = 1-\sum_{\alpha}\frac{\omega_{p\alpha}^2}{(\omega-m\omega_{\alpha})^2-(\omega_{H\alpha}+2\omega_{\alpha})^2},$$

$$\varepsilon_2 = -\sum_{\alpha}\frac{(\omega_{H\alpha}+2\omega_{\alpha})\,\omega_{p\alpha}^2}{(\omega-m\omega_{\alpha})\,[(\omega-m\omega_{\alpha})^2-(\omega_{H\alpha}+2\omega_{\alpha})^2]},$$

$$\varepsilon_3 = 1-\sum_{\alpha}\frac{\omega_{p\alpha}^2}{(\omega-m\omega_{\alpha})^2}.$$

Satisfaction of the inequalities $\mu_e \ll 1$, $|z_{ne}| \gg 1$, and $|z_i| \gg 1$, is necessary for applicability of the hydrodynamic approximation in analysis of high-frequency oscillations ($\gamma \gg \omega_{Hi}$). In addition to that, in the case of a low-density plasma, satisfaction of the inequalities $\mu_i \ll 1$ and $|z_{ni}| \gg 1$ is necessary.

Let us examine the solution of Eq. (2.1) in several cases.

a) If the transverse wavelength is considerably less than the characteristic dimensions of the plasma inhomogeneity, solution of (2.1) can be sought by the Wenzel-Kramers-Brouillon method. The, substituting expression (1.4) for ψ in (2.1), we obtain, in the local approximation, the dispersion equation

$$\varepsilon_1 + \varepsilon_3 \cos^2\Theta + \frac{1}{k_\varphi}\frac{d\varepsilon_2}{dr} = 0, \quad \cos^2\Theta \approx \frac{k_\parallel^2}{k_\varphi^2} \ll 1. \tag{2.2}$$

It follows from Eq. (2.2) that, for the unstable oscillations investigated above, approximation of a uniform plasma is applicable only at quite large values of $k_{||}$:

$$k_\parallel^2 |\varepsilon_3| \gg \left| k_\varphi \frac{d\varepsilon_3}{dr} \right|. \tag{2.3}$$

In this case, Eq. (2.2) coincides with Eq. (1.12).

In a dense plasma ($\omega_{pi} \gg |\omega_{Hi} + 2\omega_i|$), Eq. (2.2), as in the case of a uniform plasma, has a solution, with $|\mathrm{Re}\,\omega - m\omega_i|$ and γ considerably greater than ω_{Hi}. In this case, disregarding the contribution of ions to ε_2 and ε_3, we obtain [20]

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 \cos^2\Theta}{(\omega - m\omega_e)^2} - \frac{\omega_{pi}^2}{(\omega - m\omega_i)^2} - \frac{\varkappa_n \omega_{pi}^2}{k_\varphi \omega_{Hi} (\omega - m\omega_e)} = 0, \tag{2.4}$$

where /26

$$\varkappa_n = \frac{d}{dr} \ln \omega_{pi}^2.$$

By satisfying the inequality opposite in sign to (2.3),

$$k_\parallel^2 \ll \frac{\gamma}{|\omega_{He}|} |\varkappa_n k_\varphi|, \tag{2.5}$$

this equation has a solution [20]

$$\mathrm{Re}\,\omega - m\omega_i = k_\varphi u = \pm \frac{\omega_{pi}}{\sqrt{1 + \frac{\omega_{pe}^2}{\omega_{He}^2}}}, \tag{2.6}$$

$$\gamma = |k_\varphi u| \left(\frac{\varkappa_n u}{\omega_{Hi}}\right)^{1/2} \ll k_\varphi u. \tag{2.7}$$

In the case of a low-density plasma ($\omega_{pi} \lesssim \omega_{Hi}$), by satisfying an inequality opposite to (2.3), Eq. (2.4) has the form

$$1 - \frac{\omega_{pi}^2}{(\omega - m\omega_i)^2 - (\omega_{Hi} + 2\omega_i)^2} - \frac{\varkappa_n \omega_{pi}^2}{k_\varphi \omega_{Hi} (\omega - m\omega_e)} = 0. \tag{2.8}$$

The growth increment reaches a maximum value

$$\gamma_m = \omega_{pi} \left(\frac{|\varkappa_n| \omega_{pi}^2}{k_\varphi^2 u \omega_{Hi}}\right)^{1/2} \tag{2.9}$$

by satisfying the resonance condition

$$\mathrm{Re}\,\omega = m\omega_i \pm [\omega_{pi}^2 + (\omega_{Hi} + 2\omega_i)^2]^{1/2} = m\omega_e. \tag{2.10}$$

In this case, the inequality opposite to (2.3) has the form (2.5), where it should be assumed $\gamma = \gamma_m$.

b) If the rotating plasma is uniform in the region $r < a$ and its density decreases sharply to zero at $r \sim a$, Eq. (2.1) has a solution of the surface wave type, localized close to the sharp boundary $r = a$. Let us obtain the dispersion equation for surface waves.

The solution of Eq. (2.1) has the form

$$\psi = AI_m(kr) \quad (r < a),$$
$$\psi = BK_m(k_{\|} r) \quad (r > a),$$

where $k^2 = k_{\|}^2 \frac{\varepsilon_3}{\varepsilon_1}$, $I_m(x)$ is a Bessel function from the imaginary argument and $K_m(x)$ is a MacDonald function. Using the continuity conditions $\psi(r)$ at r = a and the second boundary condition

$$\varepsilon_1 \frac{d\psi}{dr}\bigg|_{r=a-0} - k_{\varphi}\varepsilon_2\psi\,|_{r=a-0} = \frac{d\psi}{dr}\bigg|_{r=a+0}, \tag{2.11}$$

which is not difficult to obtain, integrating Eq. (2.1) over a narrow region close to r = a, we obtain the following dispersion /27 equation:

$$k\varepsilon_1 I'_m(ka) - k_{\varphi}\varepsilon_2 I_m(ka) = k_{\|} K'_m(k_{\|} a) \frac{I_m(k_r a)}{K_m(k_{\|} a)}. \tag{2.12}$$

Let us note first of all that, for axially symmetric oscillations (m = 0), Eq. (2.1) coincides with the dispersion equation for symmetric waves in a plasma cylinder, in the case of a quiescent plasma and, naturally, it does not have unstable solutions.

Let us examine dispersion equation (2.12), for the case of short waves ($ka \gg 1$, $k_{\phi}a \gg 1$). In this case, we find from (2.12) that

$$\sqrt{k_{\varphi}^2 + k_{\|}^2 \frac{\varepsilon_3}{\varepsilon_1}}\,\varepsilon_1 - k_{\varphi}\varepsilon_2 + \sqrt{k_{\varphi}^2 + k_{\|}^2} = 0. \tag{2.13}$$

This equation, if substitutions (1.6) are taken into account, coincides with the dispersion equation for surface waves on a flat boundary of a magnetoactive plasma-vacuum.

In a dense plasma ($\omega_{pi} \gg \omega_{Hi}$), Eq. (2.13) has a solution with Re $\omega \gg \omega_{Hi}$ and $\gamma \gg \omega_{Hi}$. At $k_{||} = 0$, it takes the form

$$2+\frac{\omega_{pe}^2}{\omega_{He}^2}-\frac{\omega_{pi}^2}{(\omega-m\omega_i)^2}+\operatorname{sign} m\,\frac{\omega_{pi}^2}{\omega_{Hi}(\omega-m\omega_e)}=0. \tag{2.14}$$

The solution of this equation in the high-frequency region

$$\omega=m\omega_e-\frac{\operatorname{sign} m\omega_{pe}^2}{|\omega_{He}|\left(2+\frac{\omega_{pe}^2}{\omega_{He}^2}\right)} \tag{2.15}$$

corresponds to stable oscillations. One of two solutions of Eq. (2.14) in the low frequency region

$$\operatorname{Re}\omega=m\omega_i$$

is unstable, if $u = u_e - u_i > 0$, in which

$$\gamma=\sqrt{\omega_{Hi}|\,m(\omega_e-\omega_i)|}\left(|m(\omega_e-\omega_i)|\ll\frac{\omega_{pe}^2}{\omega_{Hi}\left(2+\frac{\omega_{pe}^2}{\omega_{He}^2}\right)}\right). \tag{2.16}$$

Expression (2.16) is applicable, in order of magnitude and at $k_{||} \neq 0$, if

$$k_{\|}^2 \lesssim k_\varphi^2\left|\frac{k_\varphi u}{\omega_{He}}\right|.$$

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